GEOGRAPHICAL WHOLE/PART LOGIC

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The most ancient reflections of Greek's philosophy were about our cosmos's origins and nature. They included an investigation on Earth's shape considered as a Whole as well as inhabited world's drawings made to determine the situation of its Parts with respect to the others.

Eratosthenes (-275, -193) creator the world: "geographikα" (from „gê": the earth and „grαphein": incise to write or draw), belongs this way of thinking. Although its writing disappeared, we still know that its geography was made of two books. The first one was a polemic on the geographical value of the poetic description of the world and a program to substitute it by a scientific approach. The second one was an evaluation of Earth's size (considered as a sphere), made by measuring astronomical angles and on-earth distances. Eratosthenes was then giving the first rational and geometrical picture of the world, which has been called „map" from the XVI° century in the occidental world, but whom use will only become general at the end of the XIX° century (in French, the word, "cartographe" appears only in 1877).

The history of geography is inseparable from that of cartography. Geography, description of Earth, is based on the discovery of the Earth and its always more precise representations of its surface using maps. The reciprocal assimilation of the geographical space and the cartographic space is of the order of certainty and become a fact among most of the geographers from the middle of the XX° century. Therefore, in this conception geography begins by giving the object's position using their coordinates on plane-drawn axes. This conception has the advantage of allowing the Euclidean definition of the mathematical distance (translation-invariant and symmetric) and to introduce directly the usage of geometry to represent Earth's surface in two or three dimensions. However, if classical geographers from the end of the XIX° century to the beginning of the XX° thought that geography had to be based on fore-study and systematic use of cartographic representation, most of them raised against the project consisting of reducing the geographical space's properties to those of the geometric or cartographic space. They used the All/Part thinking mechanism in order to preserve the originality of geography.

The All/Part thinking mechanism applied to the surface of the Earth, with or without the help of cartographic representation, is therefore present in the continuity of the history of Western geography, without it being necessary for it to be considered as linear and progressive. But is this a sufficient reason to make it one of the epistemological foundations of geography? Couldn't it be a reference passed on from the Renaissance to the 19th century by generations steeped in classical Greco-Latin culture, with subsequent generations trained in a modern manner no longer caring much about this relic?

We must first notice that, scientifically speaking, modern ways of thinking don't always invalidate primitive of ancient ones. Although they largely predate differential calculus and probability, arithmetic and geometry are still used in fundamental research as well as in applications. Moreover, it won't be possible to compute in geometry or probability without arithmetic. Finally, arithmetic is the first field ever used to shape what's a mathematical proof. Therefore, in scientific fields, far from becoming obsolete by further developments, older theories become cornerstone of newer ones. Consequently, unless one assumes that geography is not a science or that there is a complete break between geography before and after the end of the nineteenth century, seeking to understand and use the oldest mechanism of geographic thought is scientifically legitimate.

This being said, the All/Part thinking mechanism is not the prerogative of geographers. Modern research in experimental psychology, psychoanalysis and neurology has shown that the relationship of the part to the wholeness plays an essential role in the mental and emotional development of the child. In particular, distinguishing the Part from the Whole from the action on the totality and the elements plays a determining role in the cognitive learning of space. On the affective level, the first spatial experience of the separation of the Whole and the Part is the birth. The relationships that are established between the mother (whole) and the child (part) structure the original affectivity. The discovery of the privileged relations of the father with the mother inserts the child in a triangular situation which determines the framework of the learning of social relations. Finally, the difficulties encountered in handling totalities (All) and elements (Parts) can generate behavioral or space management problems. If they are particularly serious, they make possible disorders of the identity.

Nowadays, faced with the generality of the All/Party thinking mechanism, geographers have adopted various attitudes. Some consider it to be such a general rule that it is of only minor practical interest in geography. Others see it as an essential element of geographic thinking. Finally, some use it simultaneously with other mechanisms, because they believe that geography is a more global way of thinking. But historical research on classical (antique or modern) geographers shows that the Whole/Part mechanism is used by all of them as a tool to study relationship between earth's surface object's, in other words as a tool to study geographical objects. Whatever its status (principle, rule or minor element), the All/Party thinking mechanism is thus one of the means of accessing the truth in geography, a "logic".

The geographical definition of the whole/parts does not imply a geodesic definition or a precise geometric figuration. Nothing goes against representing a geographical object, considered as a Whole on a map's background and to interpret in a geographical way. But its geographical spatial properties do not follow from this representation. Finally, it's clear that the Whole-s may have any spatial extension, but only one has the maximal spatial extension: the Earth. Thus, the formulation presented shows that the Earth is not a metaphor that would allow to explain the properties of geographical objects considered as Wholes, but only the primitive object defined, studied and used by all geographers since antiquity to do geography. Could geography even exist without Earth being considered as a Whole?

As we will see in the remaining of this paper, the Whole/Part logic is sufficient to employ the usual geographical ways of thinking. It is therefore a starting point to which mathematical procedures must be added to allow calculation. As a result, the field of application of All/Party logic is much wider than scientific geography alone and it is, therefore, a possible way of passage between geographers and geographies, but also between geographers and non-geographers who do geography

This being the case, geography is currently literally fragmented between the geographies of professional geographers (researchers, teachers, planners, geomaticians, popularizing workers, etc.), of non-geographic professionals (cartographers, journalists, writers, politicians, ideologists, etc.), and of everyone else (popular geographies, beliefs, myths, prejudices, etc.). To try to unify them would generate new metaphors that would feed anti-scientific and political discourses. Therefore, the point of view adopted here is not to formulate a metalanguage of geographies but to try to construct a geography as an exact science, using the All/Party logic as a starting point, since it is used explicitly or implicitly by all those who "geographize" in one way or another.

The All/Party logic would thus serve two purposes. On the one hand, to create a language that would allow us to move from one geography to another, and on the other hand, to define objects, methods and techniques of calculation that are properly geographical and that would allow us to handle more rigorously the classical statistical procedures and the graphical representations: cartography, geomatics. Each geography would thus keep its own way of geographing, but by developing an autonomously logic of geography, would make it possible to move from one geography to another, to understand them in a comparative manner and to evaluate their results.

SPATIAL DIFFERENTIATION

Geography is about macroscopic objects considered on different scales. Microscopic or cosmic objects are not part of geography.

Definition 1: Is spatial any entity (L, O) with L a locus and O an object, which will never be considered individually.

Definition 2: Is geographical any information which differentiates, either the place, or the object, or the place and the object, of a spatial entity situated or located on the surface of the Earth.

If Λ is a finite set of loci and O a finite set of objects, their Cartesian product $P=\Lambda \times O$ is the set of ordered pairs $p=\langle \lambda \times o \rangle$ where λ belongs Λ and o belongs O. Two pairs $p_1 = \langle \lambda_1 \times o_1 \rangle$ and $p_2 = \langle \lambda_2 \times o_2 \rangle$ are distinct: $p_1 \neq p_2$, if there is a differentiation (written t) from at least one of its components: the locus or the object. Therefore, there are four possibilities:

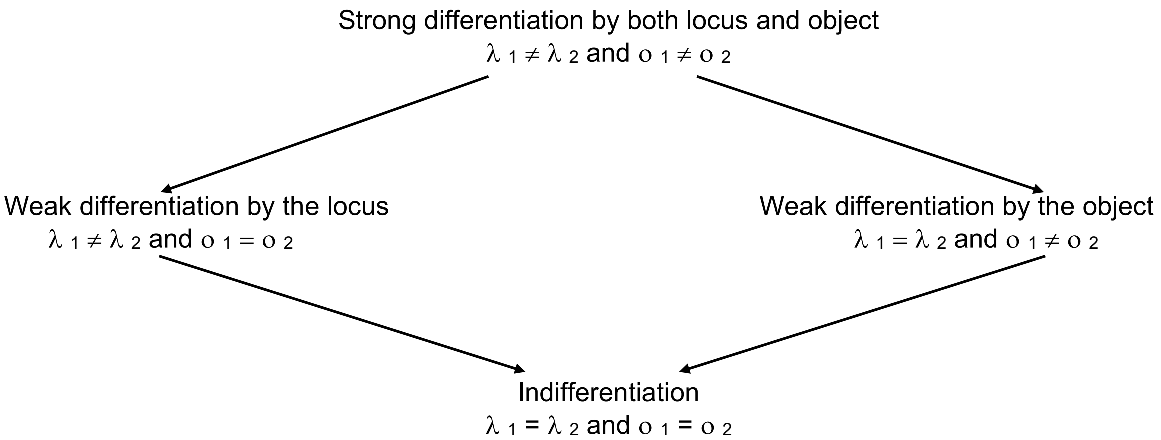
a) The differentiation comes from both the locus and the object: $\lambda_1 \neq \lambda_2$ and $o_1 \neq o_2$ (*strong differentiation*). Since the relation $\neq$ (not =) is irreflexive, symmetric and not transitive so is the strong differentiation.

b) The differentiation comes from the locus: $\lambda_1 \neq \lambda_2$ and $o_1 = o_2$ (*weak differentiation from the locus*). Using the properties of $=$ and $\neq$, one can easily see that this relation is also irreflexive, symmetric and not transitive.

c) The differentiation comes from the object: $\lambda_1 = \lambda_2$ and $o_1 \neq o_2$ (*weak differentiation from the object*). For the same reasons as in b), this relation is also irreflexive, symmetric and not transitive.

d) The in-differentiation (or equivalence): $\lambda_1 = \lambda_2$ and $o_1 = o_2$. Since the equality $=$ is reflexive, symmetric and transitive so is the in-differentiation.

Classifying from bottom the four possibilities, from the stronger to the weaker, we have:



In order to understand geographical objects, we only use a finite number of parameters to describe them. But there is always a « tension » between the visibility and the intelligibility of those descriptions, because the first does not imply the second. Therefore, cartography, one of geography's tool, has similar methods to figurative painting. In both cases, it's about representation in a bi-dimensional space. But such representation can preserve the richness of what's represented but not the shape, since there is no topological correspondence (homeomorphism) between two Euclidean spaces with different dimensions. We therefore need to embrace the limits of the cartographic representation and find ways to compensate or get above them.

Let S be the *situation*, relative position of geographical objects with respect to the others, expressed through order relations or non-metric structures, and M their geographical representation. S can be used to make an artifact M, a *geomap*.

Let L be the *location* of geographical objects through numerical coordinates and C the graphical representation of these objects. L can be used to make an artifact, a *map* C.

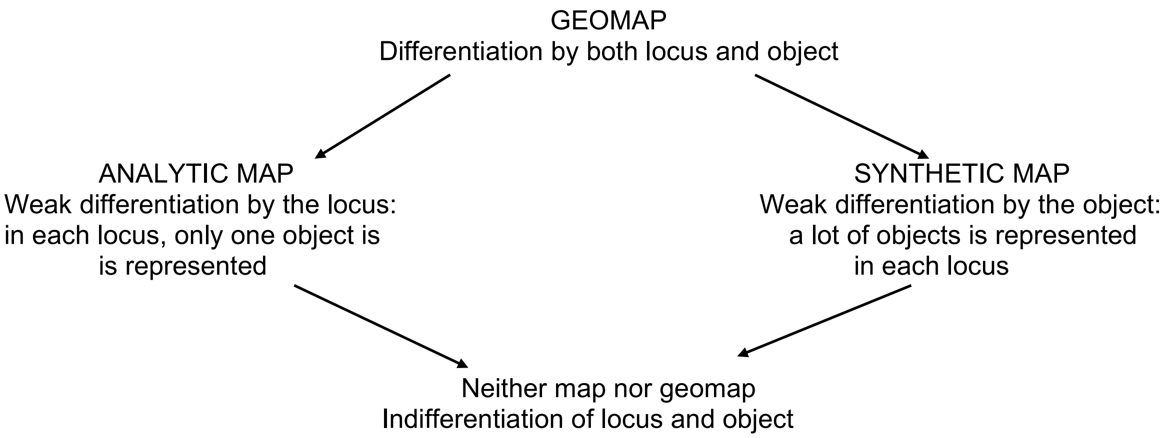
There is a duality between the geographical pair (M, S) and the cartographic pair (C, L). Each relation between M and S implies a similar one replacing M by C (resp. S by L). In other worlds geo-mapography is to cartography what localization is to situation.

On a map or a geomap relations between objects and their representations are drawn using a scale. A map's scale depends of object's sizes on one hand and their representations on the other hand. The scale of a map is a quotient of two sizes. The bigger an object's representation on a map is, the smaller its cartographic scale is.

A geomap scale on the other hand depends of the size of the geographical objects it represents. The scale of a map is the size of a quotient between two objects. This is a direct relationship: size and scale of a geomap are non-decreasingly varying. The geo-mapographic scale of big size geographical objects is big and the one of small geographical objects is small.

Then, both maps and geomaps are able to graphically represent different types of object's differentiation. The geomaps shows relations between locus-objects. It does not use localization. It can, for commodity reasons, be drawn on a map in order to emphasis for the reader the geographic space of its topic. But, since a geomap is not made to be use on the field, its drawing does not have to be as precise as a map's. The geomap is able to represent the strong differentiation by the locus and the object ($\lambda_1 \neq \lambda_2$ and $o_1 \neq o_2$), in other worlds, the identity of the geographical object.

The map, on the other hand, is based on the localization and can only represent one element of the locus-object pair. If the differentiation is due to the locus (weak differentiation: $\lambda_1 \neq \lambda_2$), then to all localization corresponds only one object ($o_1 = o_2$) and for each object to represent, we have to create a map (an *analytic map*). If, on the other case the differentiation comes from the object (weak differentiation: $o_1 \neq o_2$), all locus is indistinguishable ($\lambda_1 = \lambda_2$) and, on the same map, it's possible to represent a lot of objects on the same locus (*synthetic map*). Finally, if neither the object nor the locus of a spatial entity is differentiated, it's impossible to draw a map or a geomap for it.



DISCRETE VERSION (SET THEORETIC) OF THE WHOLE/PART LOGIC

Notations:

T: Whole: a primitive set (abbreviation W)
P: Part: a subset of the primitive set (abbreviation P)

Definitions

Let F be a finite family of geographical objects. The primitive (or order 0) geographical objects of F are those maximal for the inclusion relation. We say that a geographical object of F has order n (n>0) if it is maximal for the inclusion relation among those of F who haven't an order lower or equal to n-1.

Consider F a family whose only maximal geographical object is the Earth, considered as a set. The elements of F who are maximal subsets of the Earth are order 1 geographical objects. For example, these elements can be of two types: land and seas. It is clear that the further development of this approach will depend of the problem studied who will determine the used distinctions.

1st case: Every distinction leads to Parts P of T who could then be considered as a Whole s. Assume the distinction is made by a property. If this property leads to a precise predicate, then its associated decomposition is precise and has equivalence classes. Examples: State, province and so on or State, organization from States, from cities.

2nd case: It happens that the property is not precise. Then the decomposition of the Whole in Parts is not disjoint and that the Parts of the decomposition may not be logically interpreted: a) we come back to the first case and the decomposition in Parts is determined by an equivalence relation (reflexive, symmetric and transitive) b) Parts are coming from a relation reflexive, symmetric but not transitive we call it a tolerance relation

The same arguments as above holds for the Earth but also for any other primitive Whole.

What we presented here above is the formulation, in adequacy with on the one hand the notions, the relations and the logical operations of the Whole/Part logic and on the other hand with the set theory of the following notions generally used by geographers.

Rules of the Whole/Part logic

Rule T/P: The earth's surface is considered as a Whole, and may be split into Parts which have a spatial relation (written $*$)

$$T(A) \equiv P(A1) * P(A2) * \ldots * P(An)$$

These Parts are non-equal.

$$P(A1) \neq P(A2) \neq \ldots \neq P(An)$$

Therefore, they are spatially disjoint or have a non-empty intersection.

Equivalence rule RE: one can consider all Part on Earth as a Whole.

$$P(A2) \equiv T(B),\ P(G3) \equiv T(H),\ \ldots,\ P(K1) \equiv T(M)$$

One can divide the Wholes obtained by RE in Parts. These Wholes have the same spatial properties as the Earth.

Spatial summation rule RS*:* Any Part can be put in relation with any other Part.

$$S(A1, B3, \ldots, K2)(n) \equiv P(A1) * P(B3) * \ldots * P(K2)$$

The n above expresses the number of Parts in relation in the spatial summand.

The equivalence rule by spatial summation RES: Any spatial summand can be considered as a Whole.

$$S(A1, B3, \ldots, K2) \equiv T(W)$$

Comments on the logical formulation.

A comparative analysis of both formulations (set-theoretic and geographical) shows the following:

a) The ***T/P rule*** is expressed by giving to earth's surface the primitive object status and by considering the possible decomposition on Wholes in Parts.

b) The ∗ operation is indeed a partition, or a more general decomposition depending of the precise or un-precise nature of the considered property. The « spatially totally disjoint » case corresponds to precise properties leading to two-by-two empty intersections.

c) ***Equivalence rules RE end RES*** corresponds to the fact that the presented formulation can apply not only to Earth (the primitive whole) but also to each ulterior Whole.

d) The ***RS rule*** corresponds to the fact that when we decompose a Whole in Parts, there is a determined distinction or interference relationship between each two Parts of the decomposition.

CONCLUSION

The geographical definition of Wholes and Parts does not imply any geodesic definition nor precise geometrical formulation. Halford John Mackinder's remark about « Heartland » precisely applies to every object considered as Wholes or Parts: « The concept does not admit of precise definition on the map ». However, there is no obstruction to represent a geographical object considered as a Whole on a map background neither to interpret it on a geographical way. But this object's spatial properties are not fully given by such a representation.

This way of considering the Whole and its Parts finds his origins in Strabo on the Gaul and more specifically on one of its parts: Helvetia. Indeed, as Strabo ignored the precise description of the higher parts of Rhone's and Rhine's basin, he gave Helvetia different spatial extensions depending on the object he was considering: administrative, hydrographic, ethnic. This particularity may as well be interpreted as a scientific weakness due to lacking sufficient information, but as we found that modern authors, as classic geographers used the same kind of logic, we concluded that it was legitim and needed to be precised by formalization.

Finally, it's clear that the Whole-s may have any spatial extension, but only one has the maximal spatial extension: the Earth. The logical formulation given in this paper shows that Earth is not a metaphor used to explain properties of geographical objects considered as Whole-s but rather a primitive object defined studied and used by every geographer since antiquity in the process of doing geography.

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APPLICATION EXAMPLES

MARCUS, Solomon et NICOLAS, Georges : Exemples d'utilisation de la logique tout/partie, *in* : **Logique Tout/Partie**, *Cyberato, e-Eratosthène*, 1999
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