THE SO CALLED “CHRISTALLERIAN MODEL”

The classic formulation of the “Christallerian model” of the “theory of the centrality” is the result of a reinterpretation and a reformulation of Walter Christaller’s researches in *Die zentralen Orte in Süddeutschland* (1933).

1) This “model” is formulated in a “homogeneous” space, that is in a space where one moves in a identical way and in the same speed in all the directions (isotropy) and in which identical regular geometrical forms are deduced one from another (isomorphy).

2) This “model” works without taking into account cultural and psychological behaviors of the populations. The producers and the consumers make rational choices and move in a most economic way.

3) This “model” allows then to deduct that in “theory” the cities in which these populations live get spatially organized in hierarchical networks which work by virtue of three “principles” often called “logics”.

Market principle

This “principle” is supposed to result from economic laws of offer and demand. A city is a place of creation and consumption of wealth. It results from it a concentration, an accumulation and a convergence of population. The more a city offers of possessions and services, the more his “area of influence” as “central place” is wide. The space being homogeneous, the optimization of the distribution of cities is explained by their localization in the centers and in the summits of regular hexagonal figures. Indeed, besides itself, every “central place” situated in the center of a hexagon serves six “central places” in the summits of this hexagon. But every “central place” situated at the top of a hexagon also belongs to two other neighboring hexagons (figure 1). Consequently, for Walter Christaller the “central places” situated in 6 summits of a hexagon are served at the rate of a third by three "central places" situated in three neighboring hexagons. For a complete hexagon, the numerological coefficient of the “market principle” is thus: 1 unity for the “central place” situated in the center of the hexagon and 6 at the rate of a third for the “central places” situated in the summits, that is: $n = 1 + (6 \times ⅓) = 3$.

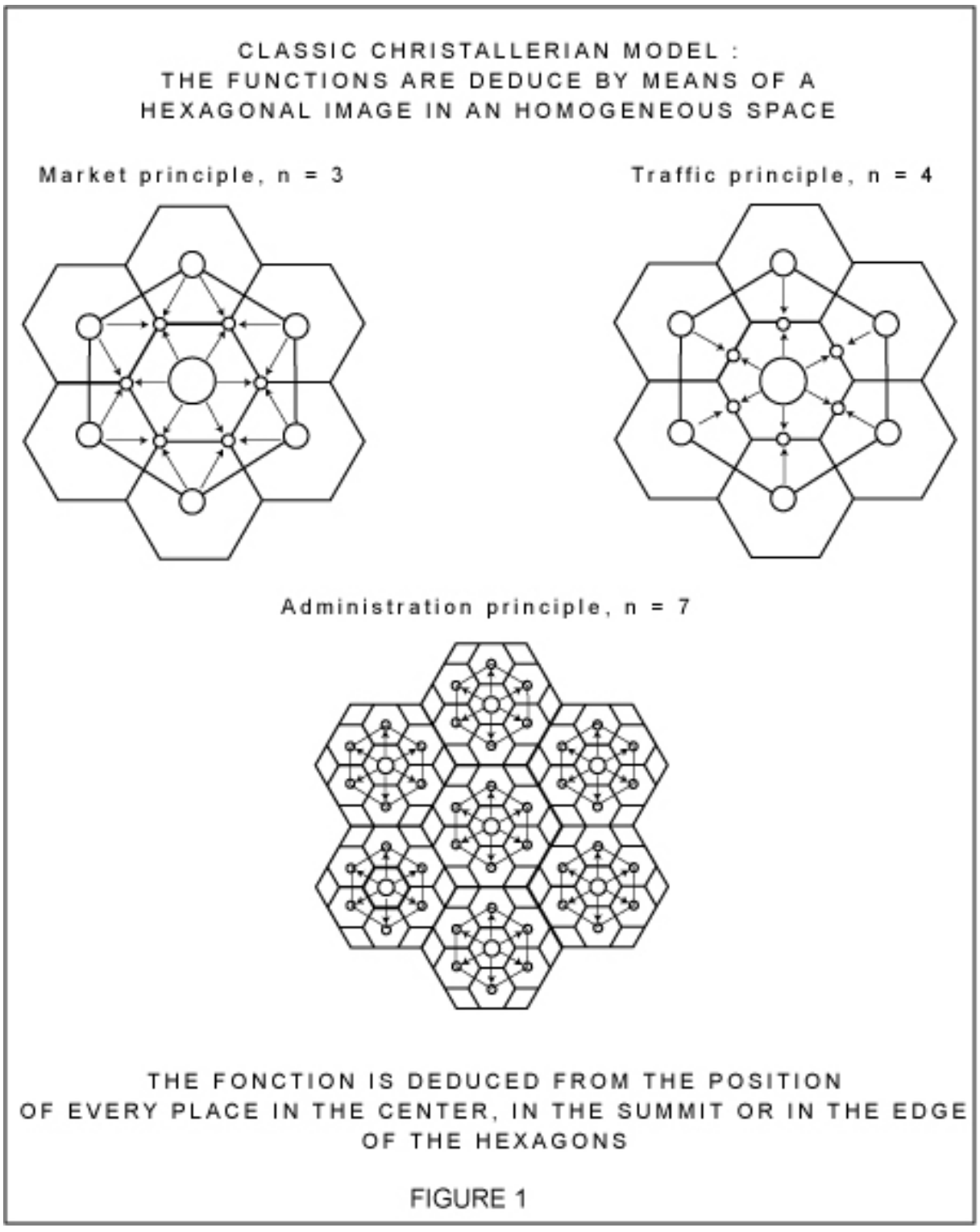
Traffic principle

This principle is supposed to result from the search for economy in the moving between the “central places”. To reduce these expenses to a minimum Walter Christaller suggests aligning the secondary “central places” between the main “central places” on the diagonals which join the centers of the initial hexagons (figure 1). Every “central place” situated in the center of a hexagon serves six “central places” situated on the edges which surround it. Inversely, every “central place” situated on one of the 6 edges of a hexagon is served for half by two “central places” localized in the hexagons neighboring to the edge where it is. For a complete hexagon, the numerological coefficient of the “traffic principle” is thus: 1 unity for the “central place” situated in the center of the hexagon and 6 times at the rate of one half for the “central places” situated on the middles of the edges : $n = 1 + (6 \times ½) = 4$.

Administration principle

This principle is supposed to result from a pyramidal spatial organization of secondary “central places” around a main “central place”. Walter Christaller places the secondary “central places” at equal distance of the main “central place” on the summits of a hexagon (figure 1). Every “central place” situated in the center of the main hexagon exercises its administrative and political power on six secondary “central places”. For a complete hexagon, the numerological coefficient of the

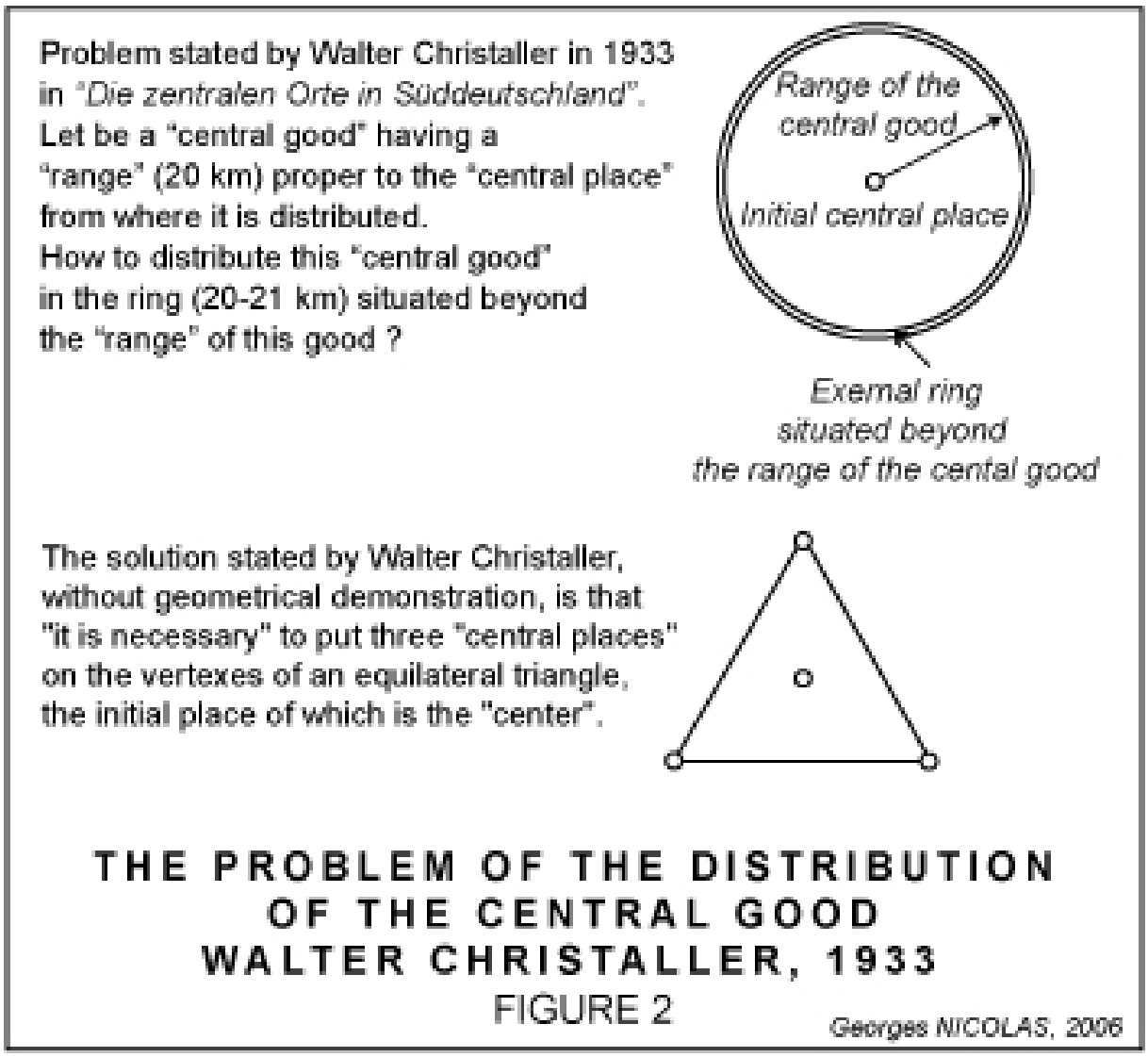
“administration principle” is thus: 1 unity for the “central place” situated in the center of the hexagon and 1 unity for every “central place” situated on the summits: n = 1 + (6 x 1) = 7.

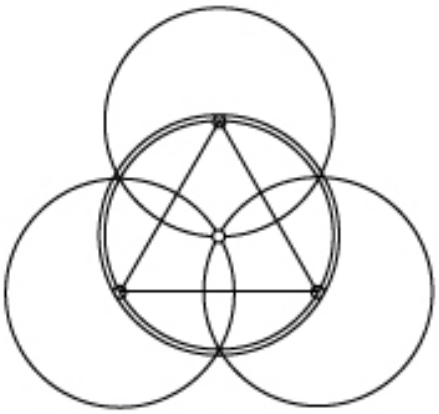


The validity of the “Christallerian model” is at the present subject to controversy in the university circles. Indeed, after having harshly been criticized before the Second World War, it was rehabilitated by the “new geographers” after the end of the conflict to be disputed again at the end of the XXth and at the beginning of the XXIth century.

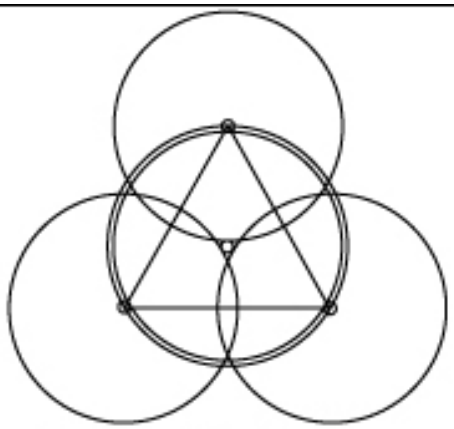
Since 1880 the mathematicians consider a “model” as a “structure which realizes the propositions of a theory” (H. Poincaré) and since 1928 the linguists see in a “model” a “simplified representation of relations between the units of a system” (V. Propp). Now, in 1933 (DS) and 1941 (DO) Walter Christaller speaks of “mathematical scheme” (“*mathematisches Schema*”) but not of “model” (*"Modell"*) of his “system of Central places” (“*System der zentralen Orte*”) in a “theory of the geography of settlements” (*"Siedlungsgeographie"* DS, p14) and not of a “theory of centrality”.

In addition, for Walter Christaller the basic geometrical figure of the system of central places is not the regular hexagon but the equilateral triangle. He takes care of reminding it by means of a drawing where he clarifies that you should not begin to think with a theoretical initial distribution of places in squares but in equilateral triangles (“*nicht die Verteilung, sondern die Verteilung*” (DS, p69)) so that places get organized in perfectly regular hexagons. This constraint is connected to the way he raises the problem and tries to resolve it geometrically.





First possibility : the new "central places" are arranged on the internal limit of the ring. One demonstrate mathematically (first error) that 1,4 % of the ring is not covered.



Second possibility : the new "central places" are arranged on the outside limit of the ring. One demonstrate mathematically (second error) that 4 % of the ring is not covered.

The geometrical solution proposed by Walter Christaller to the problem which he formulated is not "true" except for 1,4 % or 4%, or exact in 98,6 or 96 %.
It is always mathematically false an thus is not a "model".
It does not resolve the posed problem, independently of any empirical check.

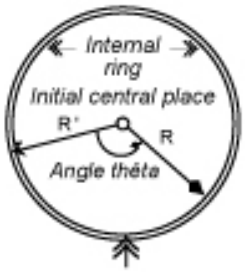
WALTER CHRISTALLER'S GEOMETRICAL ERRORS

FIGURE 3

ADAPTED FROM M.MICHALAKIS AND G. NICOLAS: *"LE CADAVRE EXQUIS DE LA CENTRALITE"*, 1986

Problem posed by Walter Christaller in 1933 in "Die zentralen Orte in Suddeutschland". Let be a "central good" having a "range" (20 km) proper to the "central place" from where it is distributed. How to distribute this "central good" in the ring (20-21 km) situated beyon the "range" of this good ?

External ring situated beyond the scope of the central good



External ring situated beyond the scope of the central good

1) The solution depends on the radius R of the internal ring, on the raduis R' of the external ring, and on the angle thêta betwen both raduises.

2) Figures are built from any point situated in the internal ring. They can have three, four, five or six edges. They have regular or irregular shape.

3) All figures can make a rotation around the center.

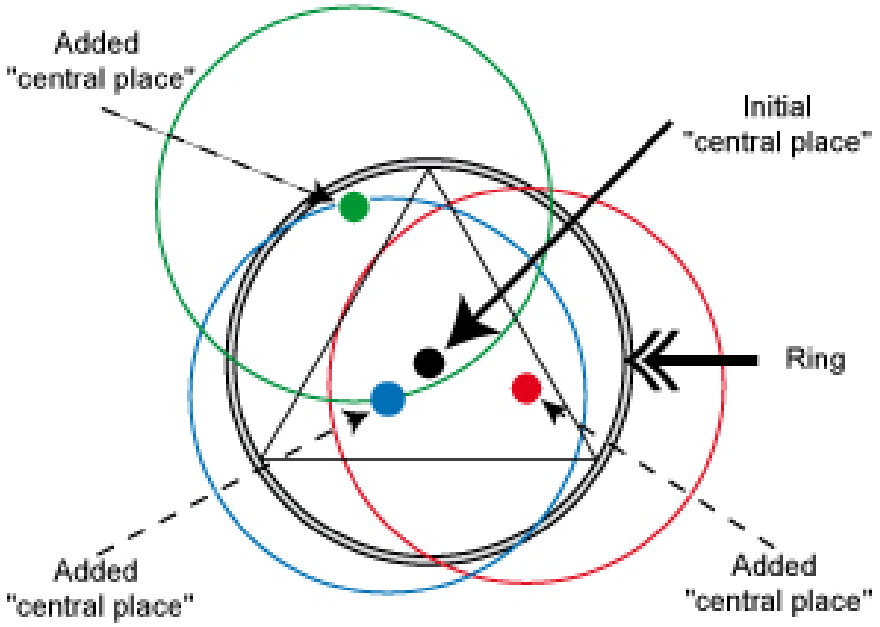
4) The number of solutions is infinite.

5) There is no normative shape.

GEOMETRICAL SOLUTION OF WALTER CHRISTALLER'S PROBLEM

FIGURE 4

ADAPTED FROM M.MICHALAKIS AND G. NICOLAS: *"LE CADAVRE EXQUIS DE LA CENTRALITE"*, 1986



Mathematically exact geometrical solution
of the "problem" of the "central place" stated by
Walter Christaller : the "central places" must be arranged
in the internal ring and not in the external ring.
All the irregular geometrical figures with
threee, four, five and six edges resolve the problem.
Represented case : figure with three edges.

Pratically there is an infinity of solutions.
The geometrical solution is thus obvious
because all the figures are possible.

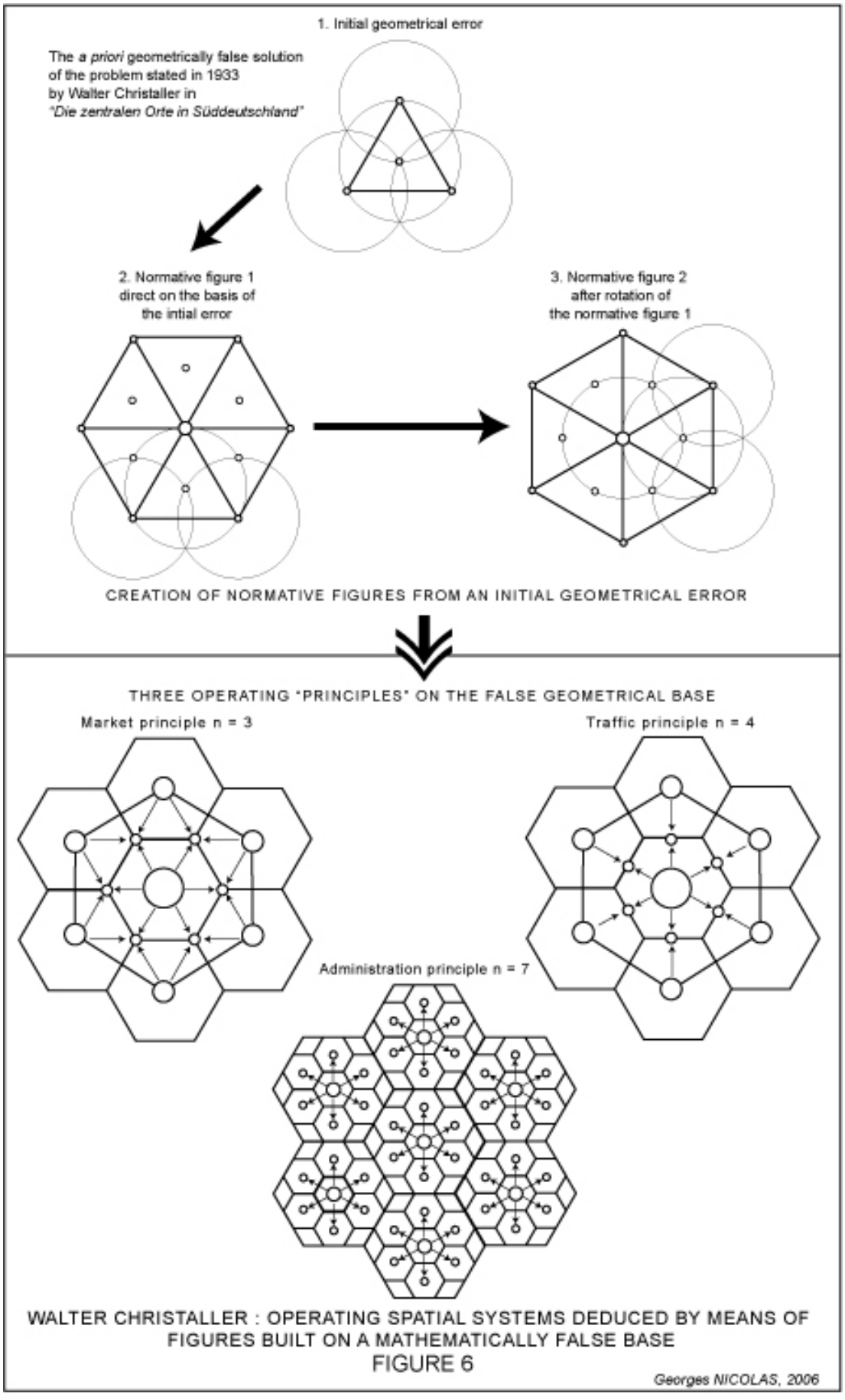
If one removes the ring by invaliding the terms
of the problem posed by Walter Christaller,
the probability to observe three places arranged
at the summit of an equilateral triangle
around an initial place situated in the center of gravity of this triangle
is 1 out of an infinity, that 0.

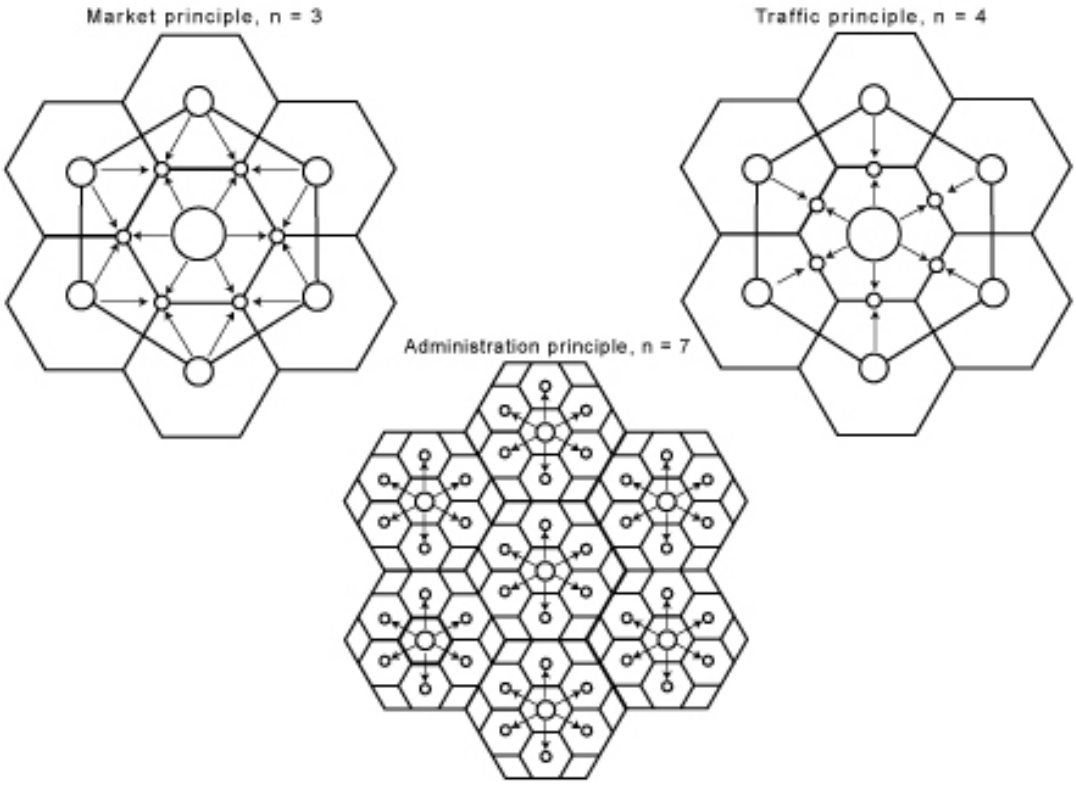
Pratically no empirical figure of this type
was thus able to be observed in the world
in all the studied historic times.

THE PROBLEM OF THE DISTRIBUTION OF THE CENTRAL GOOD STATED BY WALTER CHRISTALLER ADMITS AN INFINITY OF GEOMETRICAL SOLUTIONS

FIGURE 5

ADAPTED FROM M.MICHALAKIS AND G. NICOLAS:
"LE CADAVRE EXQUIS DE LA CENTRALITE", 1966


1. Initial geometrical error
The *a priori* geometrically false solution of the problem stated in 1933 by Walter Christaller in *"Die zentralen Orte in Süddeutschland"*
2. Normative figure 1 direct on the basis of the intial error
3. Normative figure 2 after rotation of the normative figure 1
CREATION OF NORMATIVE FIGURES FROM AN INITIAL GEOMETRICAL ERROR
THREE OPERATING "PRINCIPLES" ON THE FALSE GEOMETRICAL BASE
Market principle n = 3
Traffic principle n = 4
Administration principle n = 7
WALTER CHRISTALLER : OPERATING SPATIAL SYSTEMS DEDUCED BY MEANS OF FIGURES BUILT ON A MATHEMATICALLY FALSE BASE
FIGURE 6
Georges NICOLAS, 2006

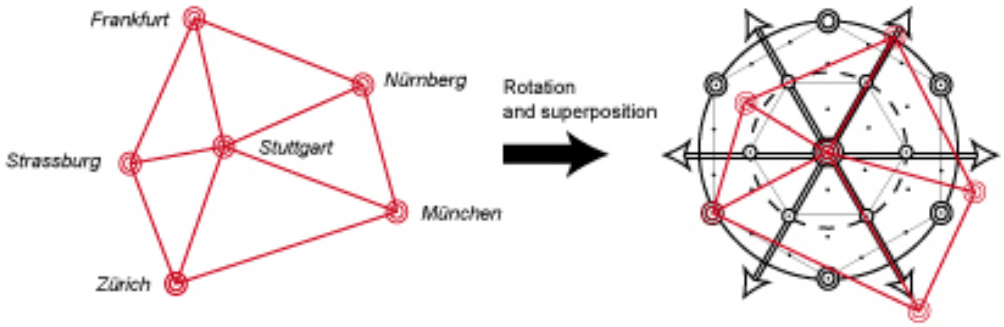


WALTER CHRISTALLER: USE OF THREE "PRINCIPLES"
TO STUDY THE INHABITED PLACES OF SOUTH GERMANY (1933)

RESULT : NEGATION OF THE REALITY OF THE OBSERVATIONS BY USING NORMATIVE FIGURES MADE FROM AN INITIAL GEOMETRICAL ERROR

Empirical figure of the L systems
resulting from Walter Chirstaller's observations
in South Germany

Normative figure inserted by Walter Christaller
into the map 5 of *"Die zentralen Orte in Süddeutschland"*
(The layouts of the hexagons do not appear on the original map)



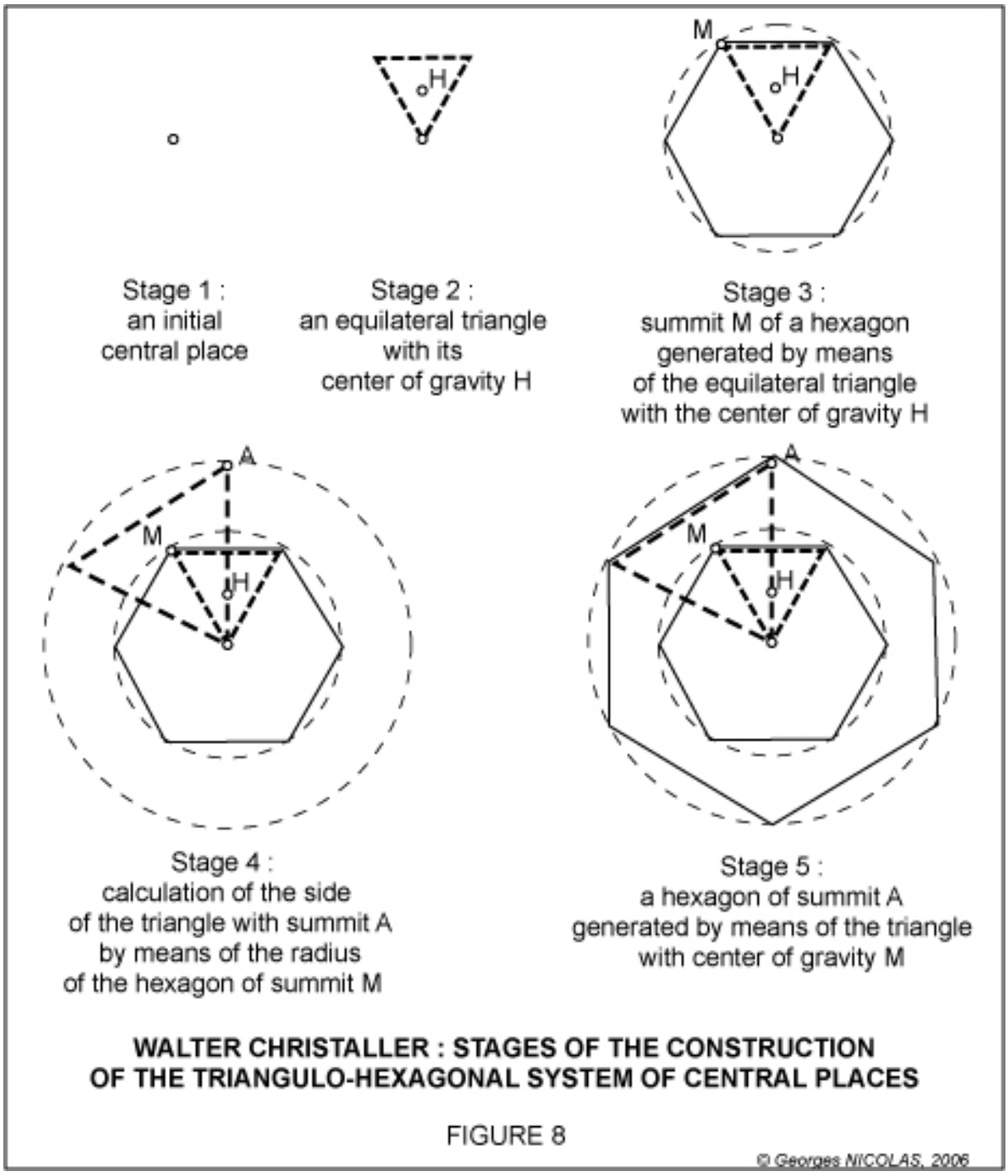
Conclusion of Walter Christaller when he compares the results (numeric and geometrical)
of the empirical figure and the normative figure of the map 5 : "what is remarquable in the observation
of the system of Stuttgart and what characterizes it in a determinating way,
it is the fact that there are 5 systems an not 6 as normally (sic)"
"*Die zentralen Orte in Süddeutschland*", p 201.

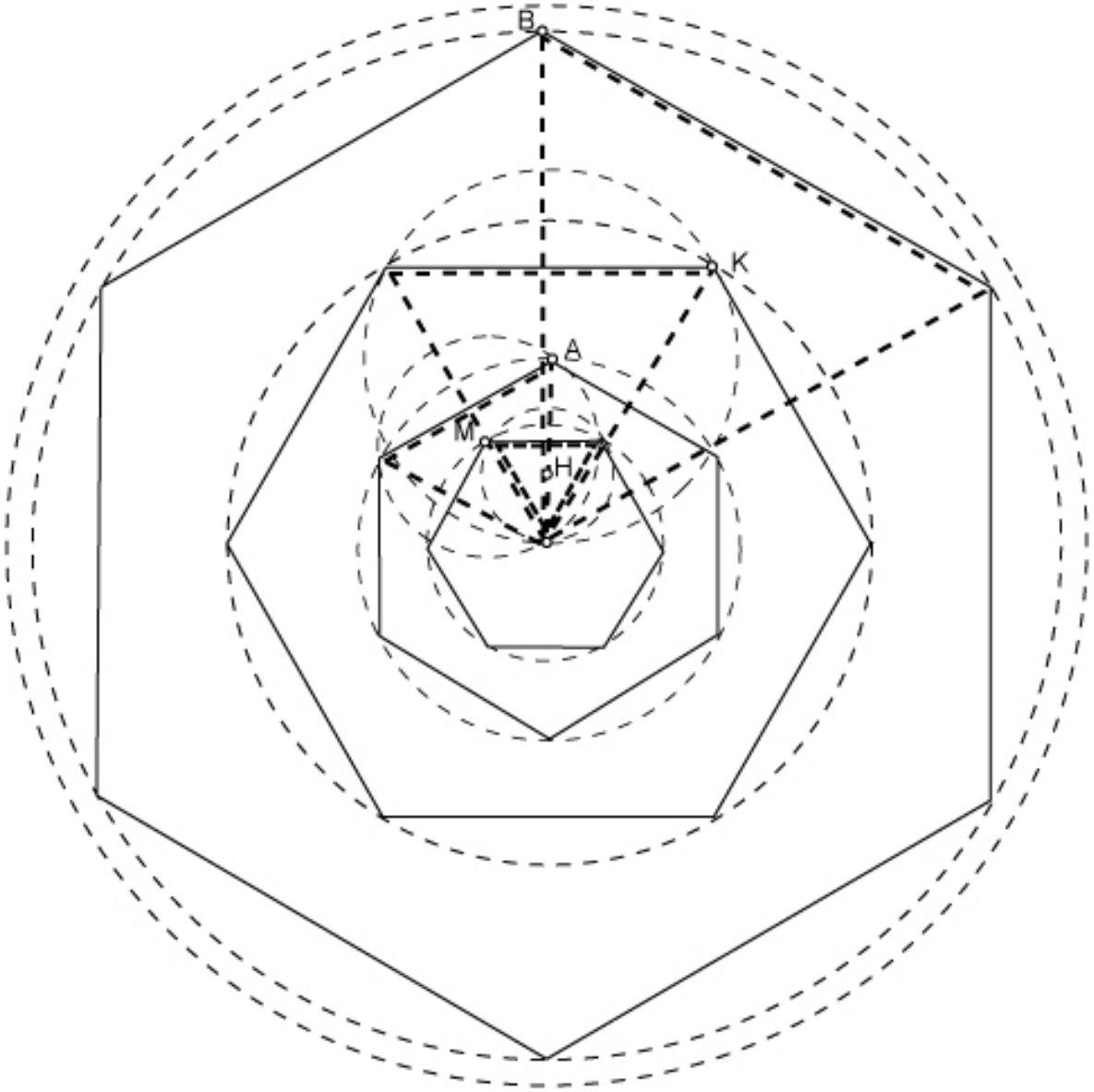
WALTER CHRISTALLER : IF IT IS CONTRADICTORY TO THE THEORY, THE REALITY IS "NOT NORMAL" ("*nicht normal*")

FIGURE 7

Georges NICOLAS, 2006

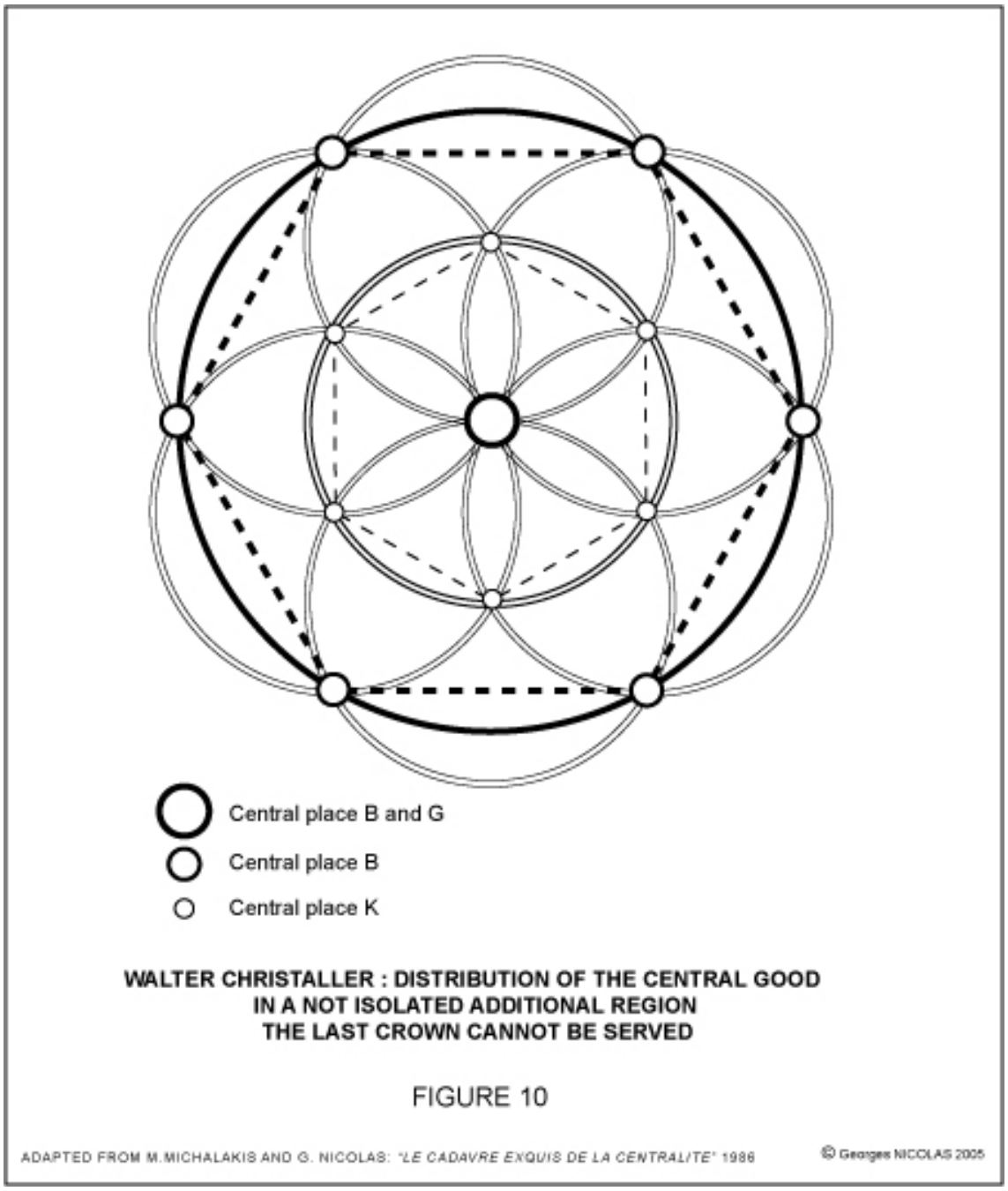
The "operating principle" deducted from the position of the "central places" in the geometrical Christallerian "schemes" want to be universal, that is valid everywhere on the surface of the Earth and functional in all the times. The "central places" are represented by means of schemes in a plan which has the same properties in all the directions: it is an isotropic space. The construction of the normative figures of the "central places" allows Walter Christaller to deduct a figure of the previous by means of the construction: equilateral triangle → regular hexagon → new equilateral triangle of superior level → regular hexagon of superior level etc. (DS p66, fig1 and CP p61, fig1): it is a isomorphous space.





WALTER CHRISTALLER :
SCHEME OF A SYSTEME OF CENTRAL PLACES
MARKET PRINCIPLE, K = 3
Die zentralen Orte in Süddeutschland Fig 1, p. 66

FIGURE 9



Walter Christaller did not manage himself to validate this representation in his researches and to find in the South of Germany in the XXth century the triangulo-hexagonal geometrical distribution planned by his theory (figure 6). Explained in a triangulo-hexagonal way, the "theory of central places" was thus invalidated for the contemporary time by Walter Christaller himself. He certainly asserted the "abnormal" character of the result of his observations in South Germany and he thus contributed to "normalize" spaces conquered in the East by the IIIth Reich. In the *Warthegau*, annexed part of western Poland, he participated in the spatial planning of the exterminations-deportations of the inhabitants in villages "to return *(abwerten)* to the typical dimension" to be able to create *(Neugründung)*, and to "develop *(entwickeln)* until the typical size" main villages of 600 inhabitants by installing settling there "native Germans". He also proposed in annexed High Silesia "to reduce in their just size" existing cities and to create a city of 450.000 inhabitants "cultural centre" […] "serving as link

between Breslau and Vienna". It doesn't matter that the theory underlying these criminal plans was scientifically erroneous: the military force, the police violence, the deportation and the extermination allowed to make a clean sweep create on which the theoretical plans could be realized.

After Walter Christaller, no geographer was able to find in the world a network of "central places" arranged in regular hexagons and obeying to the three "operating principles" deducted thanks to their positions on the triangulo-hexagonal theoretical schemes. Practically schemes are not more used than to suggest (not without difficulties!) an "idealized image" baptized "model" of the concentration of the activities in certain inhabited places qualified as "central".

However, if this scientific pseudo theory is not valid for XXth century and even less for the XXIth, why would it not be for the previous historic periods, in the modern time and in the Middle Ages, periods during which, in Europe, the inhabited places have less contrasted numbers of populations? Many historians ventured on this task by refusing to use the theory in its completeness and by choosing there what they estimated as "valid". To do it, they separated the theoretical image of its operating principles. Things being what they are, even the most convinced of the validity of what stayed of the "theory" were obliged to note three facts : 1) it is impossible to find regular hexagonal figures by drawing straight lines between the "central places" ; 2) the obtained irregular figures have very rarely six edges, but the more generally, four, five edges ; 3) these figures form cellular spaces are separated by vacuum little or not integrated into regions generated by connections between inhabited places. Walter Christaller's schemes are far too "rigid" to report "the armature" of the inhabited rural or urban cluster.

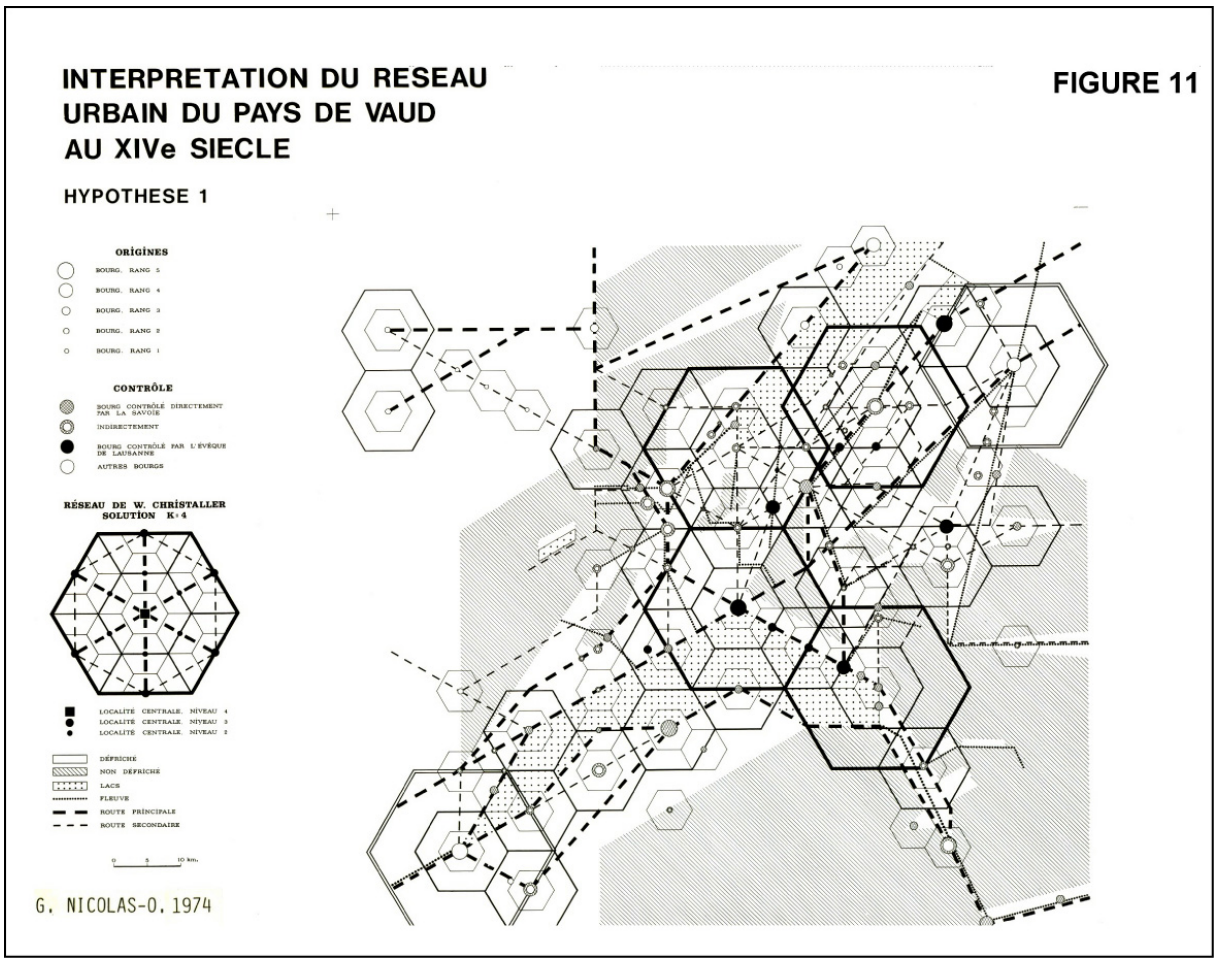
For example, a study was realized for the "burgs" of western Switzerland of the Xth to the XVth century by accepting in their completeness the theory of the central places and the triangulo-hexagonal schemes of functioning proposed by Walter Christaller.

The geometrical conception of the "system of central places" induced, to be historically used as means of analysis and representation, a harmonization between the isotropy and the isomorphy of the theoretical space and of the historic time of studied periods. The time in agreement with this type of space is isochronous (linear and divisible to infinity): it is the time of chronologies in millenniums, in centuries, in decades, years etc. The Walter Christaller's theory of central places works consequently with always identical and self-reproductive regular figures, a chronological aperiodic time and universal logics. In the case of the medieval burgs of western Switzerland the adopted chronology was that of the century.

The tested hypothesis was the following one: when a burg is in a position corresponding to an operating principle in the triangulo-hexagonal basic lattice of the space in western Switzerland in the Middle Ages (space), its rank in the hierarchy depends on its antiquity. The more the burg is ancient (time), the more its rank is high (urban hierarchy). According to Walter Christaller's project which wanted to deduct from the localization of places on its triangulo-hexagonal scheme "the law of regularity of the number, the [spatial] distribution and the size of the urban places" (DS p3, not translated by CP), the isotropic and isomorphic space and by necessity of coherence the isochronal time are independent variables which explain the urban hierarchy which becomes a dependent variable.

On the other hand, the point of departure of the geometrical representations was *a priori* in agreement with Walter Christaller's triangulo-hexagonal assertions because an irregular hexagon with edges of 12 km on average existed in the "centre" of the space studied with five burgs present during all the secular periods on the vertexes of this hexagon, the sixth vertex being occupied by a place which was not a burg. Besides, all the studied space could be reduced to a regular hexagonal general scheme with four levels built from the 12 km considered as the height of the basic hexagon made regular; the radiuses of the levels are: level 1 = 1.2 km, level 2 = 3.5 km, level 3 = 7 km, level 4 = 14 km. This favorable geometrical mechanism did not however allow to deduct the "central" distribution of burgs considered as potentially "central" in spite of the formulation of five "empirical" hypotheses only valid for the space of western Switzerland in the studied time and of two *ad hoc* hypotheses only valid for the theory of the "central places" (RB, p51-52).

Finally, the percentage of places completely thrown off center which are not at the level where the theory foresees that they should be is of 30 % at the level 1, 28 % at the level 2 and 42 % at the level 3. Only 28 % of the most important places are situated at the level 4 where the theory foresees them by virtue of the secular chronological evolution. Besides, there is not concordance between the numbers of population, the cultivated surfaces of the hexagons in which are these populations, the spatial levels and the hierarchical degrees of burgs. So, the biggest hexagon of level 4 has the biggest agricultural surface and the smallest population. Now, in the Middle Age, no country can claim to live without having a minimal agricultural activity which allows it to feed its population, except for the climatic and cyclical risks (famine, epidemics, massacres etc.).



No theory with scientific claim can stand up to such an accumulation of errors. The theory of the central places is invalidated: the data being public it is open to verify its demonstration.

The hexagonal schemes of Walter Christaller's "system of the central places" are not the "models" of a claimed "theory of the centrality".

1) The hexagonal schemes are attempts of graphic representations simplified by a theory which is refuted.

2) The hexagonal schemes are built from triangular schemes which do not resolve the problem of the centrality posed by Walter Christaller because they are geometrically false.

Scientifically there is no "Christallerian model" of the centrality.

Finally, as August Lösch showed it, it is not possible to create a theory from a false geometrical model. But August Lösch had refused to take oath to Hitler and he could not venture to criticize a Nazi

geographer who participated in the spatial planning of the conquests of the *Führer* in the East. In 1940 August Lösch what thus contented with saying carefully what he thought in a footnote drawing the attention on the erroneous character of Walter Christaller's process (RO p92-93).

In fact, their respective their thought processes are the following ones:

Walter Christaller: general principle (crystallization of a mass around a nucleus) → empirical space → triangulo-hexagonal spatial representation *a priori* → operating principles → hierarchies.

August Lösch : economic principles → mathematical formulation → geometrical forms *a posteriori* → hierarchies.

August Lösch did not extend Walter Christaller: he refuted him. It is only the superficial similarity between the drawings of hexagons at the beginning of Walter Christaller's theoretical wild imaginings and at the end August Lösch's theoretical reasoning that allowed to aggregate them wrongly. Walter Christaller's claimed "Christallerian models" have nothing to do in a theory of the centrality inspired by August Lösch's researches on "Economics of Location" ("*Die räumliche Ordnung der Wirtschaft*").

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Georges NICOLAS, January 2009